

# D2P: Distributed Dynamic Pricing Policy in Smart Grid for PHEVs Management

**Abstract**—Future large-scale deployment of plug-in hybrid electric vehicles (PHEVs) will render massive energy demand on the electric grid during peak-hours. We propose an *intelligent distributed dynamic pricing (D2P)* mechanism for the charging of PHEVs in a smart grid architecture — an effort towards optimizing the energy consumption profile of PHEVs users. Each micro-grid decides real-time dynamic price as *home-price* and *roaming-price*, depending on the supply-demand curve, to optimize its revenue. Consequently, two types of energy services are considered — home micro-grid energy, and foreign micro-grid energy. After designing the PHEVs’ mobility and battery models, the pricing policies for the *home-price* and the *roaming-price* are presented. A decision making process to implement a cost-effective charging and discharging method for PHEVs is also demonstrated based on the real-time price decided by the micro-grids. We evaluate and compare the results of distributed pricing policy with other existing centralized/distributed ones. Simulation results show that using the proposed architecture, the utility corresponding to the PHEVs increases by approximately 34% over that of the existing ones for optimal charging of PHEVs.

**Index Terms**—PHEVs, Roaming-price, Micro-grid, Vehicle to Grid (V2G), Grid to Vehicle (G2V), Smart Grid, Dynamic pricing.

## I. INTRODUCTION

Due to the major failures in traditional electric-grid, power engineers are embarking on digitalized, decentralized electric grid environment, known as smart grid, for enabling reliable, efficient, and cost-effective electric power supply to the end-users. A smart grid is characterized by a combination of an underlay electricity network, and an overlay communication network. With the growing concerns about climate change and green environments, the implementation of plug-in hybrid electric vehicles (PHEVs) is one of the most important strategies to establish green smart grid environments [1]. PHEVs are considered to be deployed in a large-scale environment to maintain adequate balance between supply and demand for energy management in smart grid. PHEVs can charge their batteries from electric grid — known as grid to vehicle (G2V), and can also discharge power to the grid — known as vehicle to grid (V2G).

### A. Motivation

PHEVs are attractive due to features such as low-cost charging and green environment (in terms of de-carbonization) [1], [2]. PHEVs can be used for outage and demand side management (DSM). Renewable energy sources (e.g., solar and wind) depend on environmental conditions. Due to this intermittent behavior of renewable energy sources, PHEVs are

considered to be useful for fulfilling the demand during peak-hours using the V2G mechanism. The integration of PHEVs into smart grid requires an intelligent charging policy, so that the energy consumption profile of PHEVs is optimized in order to have reliable, efficient, and cost-effective energy management. In smart grid architecture, typically PHEVs charge their batteries either at home during non-peak hours, or at office-premises during peak hours. In such a scenario, for large-scale deployment, PHEVs’ demands create extra loads during peak-hours on the grid, as most of the appliances are to be switched-on. Due to the increasing presence of PHEVs in a micro-grid, energy demand also increases on that micro-grid, thereby increasing the real-time *price of energy*, decided by the micro-grid. In such a condition, home customers (both residential and PHEVs) are required to pay high price for energy consumption. Additionally, grid-failure may be occurred due to the overload on the micro-grid. Therefore, an efficient and fair pricing policy needs to be designed for PHEVs charging and discharging to make a difference between the home customers and foreign customers, so as to relieve the extra load from the micro-grid during peak-hours.

### B. Contribution

In this paper, we propose an ‘intelligent’ pricing scheme for PHEVs management to address the above mentioned issues. The proposal bears analogies with the case of pricing while roaming in cellular communication. We term the concepts of home micro-grid and foreign micro-grid, as in the architecture for cellular communication. A PHEV is considered to be on the home-side, when serviced by the home micro-grid, and on the foreign-side, when serviced by other micro-grids (except the home). In such a case, a micro-grid decides real-time price dynamically, in terms of two components — *home-price* and *roaming-price* — depending on its supply-demand curve. Home-PHEV users consume energy according to the *home-price*, and foreign PHEV users charge their batteries according to the *roaming-price*, when they roam. We propose an intelligent pricing policy to evaluate an optimal real-time price. After receiving the real-time price from different micro-grids in its vicinity, a PHEV chooses a micro-grid optimally to consume energy. Consequently, after evaluating the pricing policy as *home-price* and *roaming-price*, we propose a decision making approach for selecting the optimal micro-grid to maximize the PHEVs’ utilities. We also discuss the discharging process of the PHEVs to maximize their individual utilities. On the other hand, micro-grids also optimize their revenue by implementing the distributed pricing model, so as to relieve the peak load

from the grid. In summary, the *contributions* of this work are as follows.

- We propose an ‘intelligent’ pricing policy for PHEVs charging.
- We evaluate two types of real-time price — *home-price*, and *roaming-price*. In such a pricing model, the home users consume energy with the *home-price*, and the foreign users do the same with the *roaming-price*.
- We present an algorithm for grid to evaluate real-time price, depending on *elastic* demand and supply to maximize micro-grid’s utility. The algorithm for energy exchange — charging and discharging — between the grid and the customer is also presented.
- We propose a decision making approach for optimizing the charging and discharging cost of the PHEVs to maximize the PHEVs’ utilities.

The rest of the paper is organized as follows. In Section II, we briefly present the related literature for PHEVs management. Section III describes the system model related to the problem. We propose intelligent *home-price* and *roaming-price* schemes for PHEVs charging in Section IV, and then we formulate the decision making process for energy consumption and the corresponding algorithm in Section V. The simulation results are discussed in Section VI. Finally, in Section VII, we summarize the proposed scheme with ideas for future extensions.

## II. RELATED WORKS

Several schemes exist on PHEVs charging-discharging [3]–[15]. In [3], the author proposed distributed charging methods for PHEVs. In such problems, the authors used the concept of pricing models used for Internet traffic congestion control. The smart grid decides real-time price depending on the total demand from the consumers and the total supply from renewable, and non-renewable resources. Sensor Web-services are used to control the charging strategies for PHEVs in [4]. The authors proposed two types of charging methods for the management of PHEVs charging — one is the use of gas energy, and another the use of grid energy. During peak-hours, the vehicles are charged by gas energy to relieve load from the smart grid. In such a scenario, a dynamic price is defined for all the micro-grids according to the overall load of the smart grid. In a recent study [5], a home gateway controller (HGC) is introduced for vehicle to home (V2H) energy transfer. In such a mechanism, HGC communicates with the PHEVs, and defines the charging strategy for them. In [8], the authors proposed a communication-based PHEV load management scheme (Co-PLaM). In such a scheme, the communication between charging stations and smart grid is established using the IEEE 802.11s protocol-based wireless mesh network (WMN). A substation control center (SCC) approves or rejects the charging request from the PHEVs, depending on the demand-supply curve. However, in this scenario, a centralized pricing policy is maintained, instead of the distributed one. A prediction-based charging strategy for PHEVs management is discussed by Erol-Kantarci and

Mouftah [6]. In their approach, the PHEVs receive dynamic pricing information by using wireless communication, and predicts the market price during the charging period, in such a way that the time of charging (TOC) price is low. The authors also showed that their approach works well in a cost-effective manner and provides low  $CO_2$  emissions.

The aggregated load pattern in a municipal parking deck from multiple PHEVs is studied in [16]. They also proposed the smart charging profile for PHEVs in order to allocate power optimally for PHEVs charging. On the other hand, different pricing policies are proposed in the literature in smart grid — usage-based dynamic pricing (UDP) [10], quadratic cost function (QCF) [17], [18], and distributed demand response (D2R) [3]. In [13], the authors proposed the optimal charging-discharging method for multiple PHEVs with demand side management in vehicle to building (V2B) system. They proposed two approaches for PHEVs management. First, a centralized charging-discharging method is proposed to reduce the peak demand. Second, they proposed a distributed method to encourage the PHEV users for participating in the charging-discharging process. Liang et al. proposed vehicle to grid (V2G) energy transfer as an optimal discharging mechanism for PHEVs management [19]. They used dynamic programming to model the non-stationary energy demand by the PHEVs, battery characteristics, and time of charging price. They showed that the proposed architecture works well for cost-optimization method.

*Critical analysis* of the existing works reveals that there exists research lacuna on pricing/billing policy to reduce the peak demand of PHEVs for charging-discharging their batteries. With the increase in the number of charging requests from different PHEVs, the home users (which can be residential as well as official) have to pay high price for switching-on (or for charging) their appliances (or vehicles). In this paper, we propose a distributed pricing policy for PHEVs in smart grid for reducing the peak-load in order to have a cost-optimized charging strategy for PHEVs.

## III. SYSTEM MODEL

In this Section, we present the system model for distributed pricing of the charging process in PHEVs. According to the concept of roaming in cellular communication, we propose home micro-grid and foreign micro-grid architecture in terms of PHEVs location.

Let us consider a smart grid system, in which there are  $M$  micro-grids, where  $\mathcal{M} = \{1, 2, 3, \dots, M\}$ , and  $N$  PHEVs, where  $\mathcal{N} = \{1, 2, 3, \dots, N\}$ , as shown in Figure 1(a). We consider that each micro-grid has renewable and non-renewable energy sources with self-generation capacity, and provides electricity in a certain region. A micro-grid is considered to be a home grid to a PHEV, if the PHEV is registered to that micro-grid; otherwise, it is considered as foreign to it. As stated by Mondal and Misra [20], a micro-grid can reduce and expand its service area according to the supply and demand. According to this concept, PHEVs do not always know their location, whether it is in home grid or in foreign grid. In

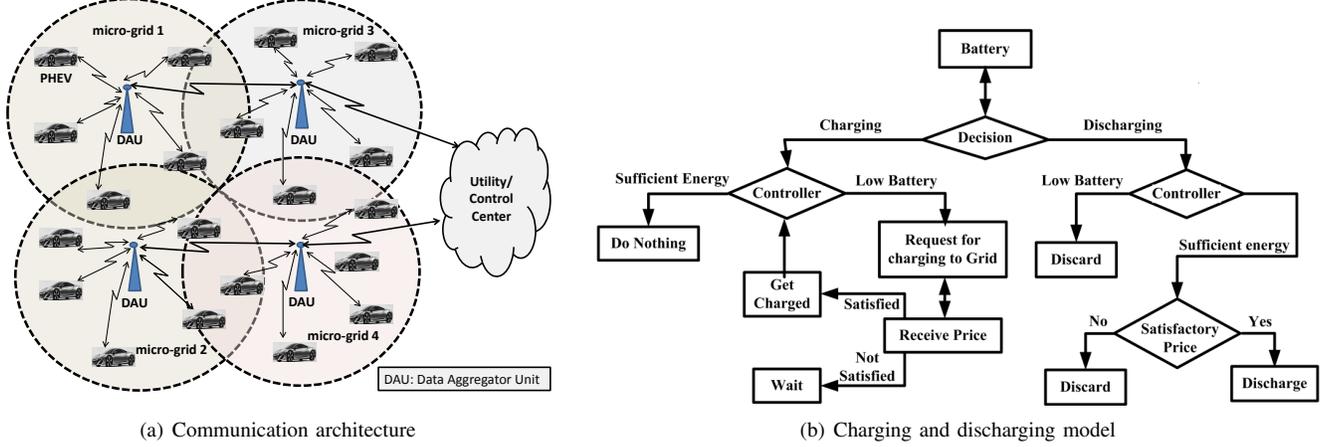


Fig. 1: Communication and charging-discharging model for PHEVs in smart grid

such a condition, we assume that each micro-grid maintains its own pricing policy depending on the total supply and demand from the customers (home and foreign). After providing the electricity to the residential consumers, the excess energy of micro-grids is used for charging the PHEVs, i.e.,

$$E_P = S_t - E_R \quad (1)$$

Equation (1) shows that excess energy,  $E_P$ , is serviced to the PHEVs for charging their batteries. However, if energy demand,  $E_R$ , from the residential customers is lower than the total supply,  $S_t$ , then PHEVs charge their batteries to maintain supply-demand curve, and, thus, obtain a well-balanced smart grid architecture. Intuitively, if supply,  $S_t$ , is lower than the demand from residential customers,  $E_R$ , then  $E_P$  has negative value, which implies that PHEVs discharge their batteries to maintain the supply-demand curve.

The mobility, battery charging-discharging, and communication models of the PHEVs are elaborated below.

#### A. PHEVs Mobility Model

We use the Gauss-Markov mobility model to design the mobility pattern of the PHEVs. According to this model, a mobile agent checks its position periodically, and updates the location whenever it reaches a threshold distance. In our work, PHEVs are considered as mobile agents, which update their locations periodically, and consequently, we use the Gauss-Markov mobility model to implement the PHEVs' mobility pattern. The velocity of PHEVs is considered to be correlated over time, i.e., the location of a PHEV at time,  $t$ , depends on its location and velocity at time,  $t - 1$ . In the proposed scenario, the movement of PHEVs is considered to be in the two-dimensional plane, and, thus, the Gaussian-Markov model is represented as [21], [22]:

$$\mathcal{V}_t = \alpha \mathcal{V}_{t-1} + (1 - \alpha) \mathcal{V} + \sigma \sqrt{1 - \alpha^2} \mathcal{W}_{t-1} \quad (2)$$

where,

(i)  $\mathcal{V}_t = [\mathcal{V}_t^x, \mathcal{V}_t^y]$  denotes the velocity of the PHEV at time  $t$ .

(ii)  $\alpha = [\alpha^x, \alpha^y]$  denotes the variance over different time.

(iii)  $\sigma = [\sigma^x, \sigma^y]$  denotes the standard deviation.

(iv)  $\mathcal{W}_{t-1} = [\mathcal{W}_{t-1}^x, \mathcal{W}_{t-1}^y]$  is the uncorrelated random Gaussian process.

From Equation (2), we evaluate the mobility model in X and Y directions, respectively, as follows:

$$\mathcal{V}_t^x = \alpha \mathcal{V}_{t-1}^x + (1 - \alpha) \mathcal{V}^x + \sigma^x \sqrt{1 - \alpha^2} \mathcal{W}_{t-1}^x \quad (3)$$

$$\mathcal{V}_t^y = \alpha \mathcal{V}_{t-1}^y + (1 - \alpha) \mathcal{V}^y + \sigma^y \sqrt{1 - \alpha^2} \mathcal{W}_{t-1}^y \quad (4)$$

To design the mobility model, we assume different locations of a PHEV, such as home, office, road, parking deck, and shopping mall with three different values of  $\alpha$  as 0, 1, and  $0 < \alpha < 1$ .

#### B. PHEVs' Battery Charging-Discharging Model

Energy demand from a PHEV depends on their available energy and battery capacity. Therefore, we consider that a PHEV can charge and discharge its battery following the condition expressed below.

$$\mathcal{X}_p^i = \begin{cases} \mathcal{C}_{PHEV}^i - \alpha_i, & \text{for charging} \\ \mathcal{B}_i - \alpha_i, & \text{for discharging} \end{cases} \quad (5)$$

where  $\mathcal{X}_p^i$  is charged or discharged energy of a PHEV,  $i \in \mathcal{N}$ ,  $\mathcal{C}_{PHEV}^i$  is the capacity of the battery,  $\mathcal{B}_i$  is the available battery energy, and  $\alpha_i$  is the battery level from where the PHEV starts charging. Additionally,

$$\mathcal{X}_P^i \leq \mathcal{C}_{PHEV}^i \quad \forall i \in \mathcal{N} \quad (6)$$

Equation (6) denotes that the charging-discharging energy of a PHEV battery is always within its battery capacity. According to [23], it is realistic to have the battery capacity of a PHEV around 30 KWh, based on current Li-ion batteries. So, we consider that PHEVs use Li-ion batteries and the energy request is between 20 KWh and 50 KWh, for simulation purpose. We show the PHEV's battery charging and discharging model in Figure 1(b).

### C. Energy Consumption Profile

We consider two types of demand — *home demand*,  $\mathcal{X}_h^t$ , and *foreign demand*,  $\mathcal{X}_r^t$ , arriving at a time interval,  $t$ . Always, total demand,  $\mathcal{X}_t$ , is real and positive, i.e.,  $\mathcal{X}_t = (\mathcal{X}_h^t + \mathcal{X}_r^t) \geq 0$ . The demand from each user can be represented in vector form as follows:

$$x_i = [x_i^1, x_i^2, \dots, x_i^t, \dots, x_i^T] \quad (7)$$

In our proposed solution, each user has a minimum and maximum energy demand, i.e.,

$$x_i^{\min} \leq x_i \leq x_i^{\max} \quad \forall i \in \mathcal{N} \quad (8)$$

The micro-grids are constrained by the supply-demand curve, i.e., the maximum load cannot exceed the total supply, so that:

$$\mathcal{X}_t \leq \mathcal{S}_t, \quad \text{where } \mathcal{S}_t = \mathcal{S}_r + \mathcal{S}_n \quad (9)$$

**Assumption 1.** *All the micro-grids can communicate with the other micro-grids, and can exchange energy with one another. If a micro-grid has excess energy, it advertises to the PHEVs to charge their batteries. In the same way, when a micro-grid has energy deficiency, it advertises to the PHEVs to discharge their batteries. For simplicity, we only consider PHEVs charging and discharging process rather than storage and energy transfer to the other micro-grids.*

### D. PHEVs and Micro-grid Communication

Real-time data from PHEVs are routed through the DAU to the control center. DAU also delivers real-time information to the PHEVs generated from micro-grids. In Figure 1(a), we show the communication model between PHEVs and control center. The DAU delivers real-time data to both parties (PHEVs and micro-grids). For this communication model, we consider the existence of some delay in gathering real-time information. The packet format of the messages for communication is shown in Figure 2.

Type (Request)	PHEV ID	PHEV Location	Requested Energy
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(a) Packet format of energy requests from a PHEV

Type (Price)	Micro-grid ID	Price (Home/Roaming)
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(b) Packet format of price generated from a micro-grid

Fig. 2: Packet format for the communication between PHEVs and micro-grids

## IV. REAL-TIME PRICING OPTIMIZATION

### A. Pricing Model

The objective of the distributed dynamic pricing is to balance the supply-demand curve for each micro-grid, in order to provide reliable energy services to the customers. Therefore, real-time price is subject to different constraints such as total supply, demand, and time. In the existing literature, real-time pricing is evaluated based only on the total supply (QCF)

[17], [18] or demand (D2R and UDP) [3], [10] or time [6]. We discuss QCF, D2R, and UDP schemes in Section VI-B in detail. We assume that the real-time price of a micro-grid is not affected by the pricing policy of the other micro-grids. We consider the real-time price as  $p_h^t$  and  $p_r^t$  for *home-price*, and *roaming-price* at time,  $t$ , respectively, for a particular micro-grid. The *home-price* and *roaming-price* are represented in vector form as follows.

$$p_h = [p_h^1, p_h^2, \dots, p_h^t, \dots, p_h^T] \quad (10)$$

$$p_r = [p_r^1, p_r^2, \dots, p_r^t, \dots, p_r^T] \quad (11)$$

The objective of a micro-grid is to maximize its own utility, depending on real-time supply and demand. Consequently, we formulate the real-time pricing model, which is decided by the grid, as a maximization problem, as follows.

$$\text{Maximize } \sum_{t \in T} \mathcal{X}_t \cdot p^t, \quad \text{where } \mathcal{X}_t = \mathcal{X}_h^t + \mathcal{X}_r^t$$

subject to

$$\sum_t \mathcal{X}_t \leq \mathcal{S}_t, \quad (12)$$

$$\mathcal{X}_t \geq 0 \quad (13)$$

Equation (12) shows that the total demand,  $\mathcal{X}_t$ , to a micro-grid should always be less than or equal to the total supply,  $\mathcal{S}_t$ , for that micro-grid. In the proposed scenario,  $\mathcal{S}_t$  is considered as the combination of grid energy and PHEVs' energy who discharge their batteries at time,  $t$ . Therefore, we consider the charging and discharging model of the PHEVs in the smart grid architecture. We assume that the total energy demand from the customers is always greater than or equal to zero, and it is represented as a constraint in Equation (13).

### B. Pricing Policy

In this subsection, we discuss the pricing policy proposed in our work. We consider two types of pricing policy — *home price* and *roaming-price*. A PHEV is registered to a particular micro-grid as its home grid, and consumes energy according to the *home price*. On the other hand, all other micro-grids are treated as foreign micro-grid to the PHEV, and consumes energy according to the *roaming-price*. *Home price* is always less than or equal to *roaming-price*. We also demonstrate the rationale behind considering *home* and *roaming* prices for PHEVs' charging, in contrast to the uniform gas price maintained for vehicles, as follows.

a) PHEVs charge their batteries either in home or in parking during office hours [6]. Consequently, increasing number of foreign PHEVs may create extra load to the micro-grid, as they charge their batteries during peak-hours (i.e., office hours). Micro-grid increases real-time price to maintain the supply-demand curve and to relieve the extra load for reliable services. Consequently, home customers are required to pay high price for energy consumption. Therefore, two different pricing mechanisms need to be followed to avoid this inequality. This

design is akin to that of cellular communication services. In the contrary, in the case of certain other utilities such as gasoline, there is much reduced chance of creating extra load due to the increasing presence of vehicles in a gasoline station.

b) PHEVs must be registered to a particular micro-grid. Therefore, hand-shaking takes place between the PHEVs and micro-grid while charging/discharging their batteries. Therefore, available bandwidth is also a crucial factor for successful communication. Consequently, foreign micro-grids provide the required bandwidth, which, in turn, bears the *roaming* concept in the smart grid architecture, as demonstrated in cellular communication. On the other hand, for gas market, situations similar to the scarcity of bandwidth occur more infrequently.

c) In the gas market, vehicles are refilled from a refilling station where fuel is stored. Therefore, the refilling station does not have any problem of real-time supply-demand management. On the other hand, micro-grids service the customers depending on real-time supply and demand. They cannot service the customers in non-real time basis. Therefore, to maintain the balance between supply-demand curve and a fair pricing policy, we need to consider different pricing policies for PHEVs management in smart grid.

### C. Pricing Optimization

We set a fixed base-price.  $p_b$ , for consuming an unit of electricity. In this work, we assume that all the micro-grids maintain a uniform base-price. We solve the optimization problem stated in Section IV-A using the linear optimization approach while considering all the constraints. Each micro-grid evaluates the real-time price,  $p^t$ , depending on the real-time energy supply,  $\mathcal{S}_t$ , and demand,  $\mathcal{X}_t$ , to the micro-grid, and it is represented as follows.

$$p^t = p_b + \{\tan^{-1}(e^\lambda) - \gamma\} \quad (14)$$

In Equation (14),  $\lambda$  is the difference between the energy demand,  $\mathcal{X}_t$ , and supply  $\mathcal{S}_t$ , and can be written as  $\lambda = (\mathcal{X}_h^t + \mathcal{X}_r^t) - \mathcal{S}_t$ , and  $\gamma$  is a pre-determined constant. From Equation (14), we see that with an increase in the supply,  $\mathcal{S}_t$ , real-time price,  $p^t$ , decreases while demand,  $\mathcal{X}_t$ , from the customers is either fixed or decreases. On the other hand, the real-time price increases with an increase in the demand,  $\mathcal{X}_t$ , from the customers while supply,  $\mathcal{S}_t$ , is either fixed or decreases. Consequently, customers are interested to consume more energy when the price is low, and vice-versa, depending on the real-time price. Therefore, Equation (14) illustrates a well-balanced pricing scheme in order to maintain the supply-demand curve while considering the customers' participation.

We consider that the *home-price*,  $p_h^t$ , is the same as the real-time price,  $p^t$ . Thus, the *home-price* is represented as:

$$p_h^t = p^t \quad (15)$$

After evaluating the *home-price* for home users, we determine the *roaming-price* while considering the pricing policy discussed in Section IV-B. We consider the *roaming-price* as:

$$p_r^t = p_h^t + [a(\mathcal{X}_r^t)^2 + b\mathcal{X}_r^t + c] \quad (16)$$

In Equation (16),  $a$ ,  $b$ ,  $c$  are predefined constants, where  $a \geq 0$ ,  $b \geq 0$ , and  $c \geq 0$ . The *roaming-price* depends on the *home-price*, and the total foreign demands from the foreign users as well. From Equation (16), we see that the *roaming-price* is always greater or equal to the *home-price*, as defined in Section IV-B, with the predefined constants. The *roaming-price* is equal to the *home-price* when all the predefined constants are zero. On the other hand, the *roaming-price* is always greater than the *home-price* when at-least one of the predefined constant is greater than zero with a demand from foreign customers.

**Assumption 2.** We assume that a micro-grid decides the real-time price based on a base-price. The base price is defined in ideal condition, when demand and supply are the same. We do not consider any electricity loss due to transmission.

### D. Algorithm for micro-grid

We show the energy exchange between micro-grid and PHEVs in Algorithm 1. The time complexity of the proposed algorithm for the micro-grids is represented as  $O(1)$ .

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#### Algorithm 1: Algorithm for micro-grid

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**Input:** Total supply,  $\mathcal{S}_t$ , Total home demand,  $\mathcal{X}_h^t$ , Total foreign demand,  $\mathcal{X}_r^t$   
**Output:** Real-time pricing as *home-price*,  $p_h^t$ , and *roaming-price*,  $p_r^t$

- 1 **if**  $(\mathcal{X}_h^t + \mathcal{X}_r^t) \leq \mathcal{S}_t$  **then**
- 2     Calculate real-time price,  $p^t$ , as in Equation (14);
- 3     Calculate the *home-price*,  $p_h^t$ , and the *roaming-price*,  $p_r^t$ , from Equations (15) and (16), respectively;
- 4     Broadcast  $p_h^t$ , and  $p_r^t$  to the customers;
- 5     Receive the confirmation from customer (PHEVs);
- 6     **if** PHEV is in home-side **then**
- 7         Exchange energy according to *home-price*,  $p_h^t$ ;
- 8     **else**
- 9         Exchange energy according to *roaming-price*,  $p_r^t$ ;
- 10 **else**
- 11     Reply with a message to wait until  $(\mathcal{X}_h^t + \mathcal{X}_r^t) \leq \mathcal{S}_t$ ;
- 12     Requests PHEVs to discharge their energy to balance supply-demand curve;

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## V. DECISION MAKING FOR ENERGY CONSUMPTION

In this Section, we define the utility for PHEVs with consumed energy, real-time price, and distance to the micro-grid from where the PHEVs charge and discharge their batteries. Consequently, we discuss the decision making process of the PHEVs for charging and discharging their batteries. PHEVs take decisions according to the available energy, distance to the charging-discharging station, and real-time price.

### A. PHEVs' Utility

The utility function of the PHEVs,  $\mathcal{U}_P$ , depends on the various parameters (such as  $\mathcal{X}_P$ ,  $p^t$ ,  $D_c^j$ ), and it is expressed as follows:

$$\frac{\partial \mathcal{U}_P(\mathcal{X}_P^t, p^t, p_b, D_c^j)}{\partial \mathcal{X}_P} = \begin{cases} \leq 0, & \text{for charging} \\ \geq 0, & \text{for discharging} \end{cases} \quad (17)$$

$$\frac{\partial \mathcal{U}_P(\mathcal{X}_P^t, p^t, p_b, D_c^j)}{\partial p^t} = \begin{cases} < 0, & \text{for charging} \\ > 0, & \text{for discharging} \end{cases} \quad (18)$$

$$\frac{\partial \mathcal{U}_P(\mathcal{X}_P^t, p^t, p_b, D_c^j)}{\partial D_c^j} = \begin{cases} < 0, & \text{for charging} \\ < 0, & \text{for discharging} \end{cases} \quad (19)$$

where  $\mathcal{X}_P^t$ ,  $p^t$ , and  $D_c^j$  are charged/discharged energy, real-time price, and distance to the charging/discharging station, respectively.

### B. Decision Making Process

PHEVs always try to minimize the charging cost,  $\mathcal{C}_t$ , and, thus, maximize their utility. In the proposed architecture, all PHEVs have full information about all micro-grids with real-time price in terms of *home-price* and *roaming-price*. PHEV takes decision considering real-time prices received from all micro-grids. Therefore, the decision making process of a PHEV considers the global information available from the micro-grids. We use a multi-attribute decision making methodology to take optimal decision, as explained below.

Let there be  $j$  micro-grids in a particular PHEV's vicinity. Then, the decision matrix, which is based on several decision parameters, is represented as follows:

$$\begin{matrix} & M_1 & \dots & M_j \\ D_c & \begin{pmatrix} D_c^1 & \dots & D_c^j \end{pmatrix} \\ p_h^t & \begin{pmatrix} p_{h1}^t & \dots & p_{hj}^t \end{pmatrix} \\ p_r^t & \begin{pmatrix} p_{r1}^t & \dots & p_{rj}^t \end{pmatrix} \\ \mathcal{X}_P^t & \begin{pmatrix} \mathcal{X}_P^1 & \dots & \mathcal{X}_P^j \end{pmatrix} \\ \mathcal{B} & \begin{pmatrix} \mathcal{B}^1 & \dots & \mathcal{B}^j \end{pmatrix} \\ \mathcal{C}_d & \begin{pmatrix} \mathcal{C}_d^1 & \dots & \mathcal{C}_d^j \end{pmatrix} \end{matrix}$$

**Assumption 3.** The real-time prices,  $p_h^t$ , and  $p_r^t$ , remain unchanged between the time of taking decision and end of charging the battery.

So, the decision rule,  $\zeta(y)$ , with full information, can be written as:

$$\zeta(y) = \operatorname{argmin}(y, M_j) \quad (20)$$

where  $M_j$  is the selected micro-grid from which a PHEV,  $j$ , consumes energy,  $M \in \mathcal{M}$  and  $j \in \mathcal{N}$ , and  $y$  is the set of full information, i.e.,

$$y = \{D_c, p_h^t, p_r^t, \mathcal{X}_P^t, \mathcal{B}, \mathcal{C}_d\}$$

According to Equation (20), the charging cost for a PHEV,

$j \in \mathcal{N}$ , for micro-grids,  $i \in \mathcal{M}$ , is determined as follows:

$$\mathcal{C}_j(i) = \begin{cases} \mathcal{C}_h(i) = (D_c^i \mathcal{C}_\beta + p_{hi}^t \mathcal{X}_{Pi}^t), & \text{for home-price} \\ \mathcal{C}_r(i) = (D_c^i \mathcal{C}_\beta + p_{ri}^t \mathcal{X}_{Pi}^t), & \text{for roaming-price} \end{cases} \quad (21)$$

From Equation (21), we determine a minimum value to maximize the pay-off. Therefore, Equation (21) reduces to

$$\mathcal{C}_t = \min_{j \in \mathcal{N}} \mathcal{C}_j(i) \quad (22)$$

**Theorem 1.** There exists a minimum value of the function  $\mathcal{C}_j(i)$  in Equation (21), for any  $j \in \mathcal{N}$ .

*Proof:* Let there exists a minimum value of  $\mathcal{C}_j(i)$ . Then, the following condition must be satisfied.

$$\min \mathcal{C}_j(i) = \min(\mathcal{C}_h(a), \mathcal{C}_r(b)), \text{ where } a, b \in i, i \in \mathcal{M}, j \in \mathcal{N}$$

Therefore,  $\mathcal{C}_j$  has a minimum value if there exists minimum value for  $\mathcal{C}_h(i)$  and  $\mathcal{C}_r(i)$ . It is obvious that there exists a minimum value for  $\mathcal{C}_r(i)$  as it is an increasing function of  $i$ . Again, in the similar manner for  $\mathcal{C}_h(i)$  as well, there must exist a minimum value. So, we get a minimum value from  $\mathcal{C}_h(i)$  and  $\mathcal{C}_r(i)$ . Therefore, if we have a value for all parameters mentioned in Equation (21), it is obvious that there exists a minimum value for  $\mathcal{C}_j(i)$ . ■

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### Algorithm 2: Algorithm for PHEVs charging/discharging

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**Input:** Required energy,  $\mathcal{X}_P^t$ , Number of micro-grids,  $j$ , Available energy,  $\mathcal{B}$

**Output:** Total cost,  $\mathcal{C}_t$ , to charge the battery of the PHEV

- 1 Requests energy with ID,  $PH_{ID}$ , and location,  $PH_{Loc}$ ;
- 2 Calculate the distance,  $D_\psi$ , the PHEV can travel with  $\mathcal{B}$ ;
- 3 **for**  $m = 1$  **to**  $j$  **do**
- 4     Receive  $p_{hj}^t$  and  $p_{rj}^t$ , from micro-grid,  $j \in \mathcal{M}$ ;
- 5     Calculate distance,  $D_c^j$ , to each charging station,  $j$ ;
- 6     Calculate cost,  $\mathcal{C}_d^j$ , to travel to charging station,  $j$ ;
- 7     **Charging;**
- 8     **for**  $k = 1$  **to**  $j$  **do**
- 9         **if**  $(D_c^j \leq D_\psi)$  **then**
- 10             Calculate total cost,  $\mathcal{C}_j$ , to charge from micro-grid,  $j$ , from the Equation (20);
- 11     Choose the optimal cost,  $\mathcal{C}_t$ , to maximize the utility;
- 12     **Discharging;**
- 13     **for**  $k = 1$  **to**  $j$  **do**
- 14         **if**  $\mathcal{B} p_j^t > (\mathcal{B} p^t + \mathcal{C}_d^j)$  **then**
- 15             Calculate  $\mathcal{U}_P^j$  for discharging to micro-grid,  $j$ ;
- 16 Select the micro-grid,  $j$ , for which utility is maximized;
- 17 Discharge energy to the micro-grid,  $j \in \mathcal{M}$ ;

---

**Assumption 4.** PHEVs take decision after gathering all information during a time interval,  $t$ , as full information is better than partial one. Where we consider the full information as real-time price information from all micro-grids, and partial

information as real-time price information from some of the micro-grids, rather than from all of them.

**Theorem 2.** *Utility with full information is greater than or equal to that of the partial one, i.e., if  $y = (y_1, y_2)$ , then with probability 1, we have  $E\{\mathcal{V}(y_1, y_2)|y_j\} \geq \mathcal{V}(y_j)$ , where  $y$  is the set of information.*

*Proof:* From the decision rule with information [24], we get  $\mathcal{V}(y_j) = \max_{a \in A} E\{\mathcal{U}(\mathcal{Z}^*, a)|y_j\}$

$$= \max E\{E\{\mathcal{U}(\mathcal{Z}^*, a)|y_1, y_2\}|y_j\} \quad (23)$$

From Equation (23), we have:

$$\begin{aligned} & \max E\{E\{\mathcal{U}(\mathcal{Z}^*, a)|y_1, y_2\}|y_j\} \\ & \leq E\{\max_{a \in A}\{\mathcal{U}(\mathcal{Z}^*, a)|y_1, y_2\}|y_j\} \quad (24) \\ & = E(\mathcal{V}(y_1, y_2)|y_j) \end{aligned}$$

From Equations (23) and (24), we see that utility with full information is greater than or equal to that with partial one, i.e.,  $E\{\mathcal{V}(y_1, y_2)|y_j\} \geq \mathcal{V}(y_j)$ . ■

#### C. Algorithm for PHEV

We show the charging and discharging process of a PHEV, as presented in Algorithm 2. The time complexity of the proposed charging/discharging process is  $O(n^2)$  for both cases — charging and discharging.

#### D. Discussion

It is important to note that the objective of the micro-grid is to determine the real-time price of energy. The corresponding objective function is presented accordingly in Section IV-A. Therefore, as presented in the same section, we have a maximization problem for the micro-grid, as the micro-grids always want to maximize their utility. On the other hand, the PHEVs take optimal decisions based on the decision parameters presented in the form of a decision matrix to minimize the charging cost, as explained in Section V-B.

While Algorithm 1 is based on the objective function of the micro-grid described in Section IV-A, Algorithm 2 is based on the decision matrix explained in Section V-B. A micro-grid does not broadcast its real-time price to the customers, while the total demand is greater than or equal to the total supply. On the other hand, after receiving the real-time prices from different micro-grids, the PHEVs choose the optimal ones for which their charging-cost/discharging-profit is minimized/maximized.

## VI. PERFORMANCE EVALUATION

### A. Simulation Settings

To simulate the overall scenario, we use NS-3 (<http://www.nsnam.org>). The demand from the PHEVs are chosen according to their battery condition, as discussed in Section III-B. In Table I, we show the various parameters used for simulation. The parameter, *energy cost*, denotes the cost for energy supply to the micro-grids.

TABLE I: Simulation Parameters

Parameter	Value
Simulation area	2 Km × 2 Km
Number of micro-grids	5
Number of PHEVs	50
Simulation time	720 sec
PHEVs min. charged/discharged energy	20 KWh
PHEVs max. charged/discharged energy	50 KWh
Base price ( $p_b$ )	10 Cents/KWh
Energy cost to the micro-grid	5 Cents/KWh

### B. Benchmarks

The performance of the proposed distributed dynamic pricing policy (D2P) is evaluated by comparing it with other pricing policies, such as distributed demand response (D2R) [3], usage-based dynamic pricing (UDP) [10], quadratic cost function (QCF) [17], [18] for real-time pricing, and static pricing policy. In the proposed scheme (D2P), a decentralized distributed pricing policy is evaluated. We refer to these different pricing policies as D2P, D2R, UDP, and QCF through the rest of the paper.

In UDP, real-time pricing is considered as  $f(x_{i,t}) = a + bx_{i,t} + cx_{i,t}^2$  during peak hours, while demand is greater than a threshold value  $x_a$ , where  $x_{i,t}$  is the energy demand, and  $a$ ,  $b$ , and  $c$  are predefined constants. Otherwise, the real-time price is fixed. On the other hand, D2R is proposed as  $f(x_{i,t}) = ax_{i,t}^k$ , where  $a$  and  $k$  are also predefined constants. In such pricing models, the costs incurred by the customers directly depend on the demanded energy even-though micro-grids have excess energy to serve. As a result, customers may not be interested to consume more energy, and, thus, excess energy may be useless. QCF is defined as  $f(\mathcal{S}_t) = a\mathcal{S}_t^2 + b\mathcal{S}_t + c$ , where  $a$ ,  $b$ , and  $c$  are predefined constants. In such a pricing model, the grid decides the price depending on the supply to it. As a result, customers may have to pay more price, even though the total demand is low.

On the other hand, D2P evaluates real-time price according to the difference between total supply and demand to the grid to maximize the grid's revenue as well as to maintain a fair pricing policy. In a centralized pricing model, real-time pricing is determined by the central grid depending on the real-time supply or demand to the grid. On the other hand, decentralized approach discusses that price is determined by the central grid, and several task are maintained by the micro-grids. Similarly, when price and task are performed by the micro-grids independently, the approach is called as distributed. In D2P, the micro-grids decide the real-time price based on individual real-time supply and demand. Each micro-grid provides services to the customers in a particular region, and real-time price is not affected by the price maintained by the other micro-grids. Therefore, the proposed scheme is modeled in a distributed and decentralized manner. On the other hand, D2R, UDP, and QCF illustrate a distributed demand response, usage-based pricing scheme, and quadratic

cost function, respectively.

### C. Performance Metrics

- *Real-time Price*: The real-time price is evaluated from Equation (14) as *home* and *roaming* price according to Equations (15) and (16), respectively.
- *Charging Delay*: We calculate the delay for charging the PHEVs. The delay is the combination of propagation delay, and the time to travel to the selected micro-grid's charging station, and can be expressed as:

$$\delta = \eta + \frac{D_c^j}{\mathcal{V}_t} \quad (25)$$

where,  $\eta$  is the propagation delay, and  $\mathcal{V}_t$  is the PHEV's velocity per second at time,  $t$ .

- *PHEV Charging Cost*: The PHEVs charging cost is calculated according to Equation (20). The cost is proportionally related to the real-time price,  $p^t$ , and distance,  $D_c^j$  to the charging station of the selected micro-grid,  $j$ .
- *Utility of PHEVs*:
  - a) *Utility for Charging*: PHEVs take decisions based on the parameters as discussed in Section V-A. We calculate the utility of the PHEVs as charging cost differences using D2R, UDP, QCF, and static pricing policy than that of using D2P. Thus, we denote the utility function corresponding to a PHEV as follows:

$$U_P(\mathcal{X}_P^t, p^t, D_c^j) = \begin{cases} \mathcal{X}_P^t(p_d - p^t), & \text{for D2R} \\ \mathcal{X}_P^t(p_u - p^t), & \text{for UDP} \\ \mathcal{X}_P^t(p_q - p^t), & \text{for QCF} \\ \mathcal{X}_P^t(p_b - p^t), & \text{for static} \end{cases} \quad (26)$$

where  $p_d$ ,  $p_u$ , and  $p_q$  denote the real-time prices obtained using D2R, UDP, and QCF pricing policy, respectively.

b) *Utility for Discharging*: We calculate the utility for discharging energy as the difference between previous charging cost and the cost gained by discharging using real-time price. Mathematically,

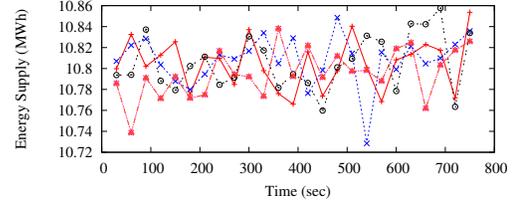
$$U_P(\mathcal{X}_P^t, p^t, p^\tau) = \mathcal{X}_P^t(p^t - p^\tau), \text{ where } \tau \in T \quad (27)$$

where  $\mathcal{X}_P^t$ ,  $p^t$ ,  $p^\tau$  are the amount of discharged energy, real-time price, and charging price, respectively.

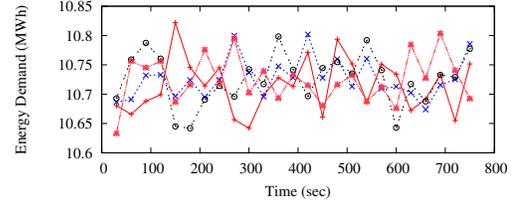
### D. Results and Discussion

We determine the base-price according to the US energy report<sup>2</sup>. For simulation, we assume that each micro-grid calculates the real-time supply and demand in every 3 seconds interval. In Figure 3, we show the supply and demand to the micro-grids, individually.

In Figure 4(a), we show the changes in real-time *home-price*,  $p_h^t$ , according to the supply and demand to the micro-grids, as shown in Figure 3. We see that sometimes the *home-price* is greater than 10, when the demand is greater than the



(a) Real-time supply for micro-grids



(b) Real-time demand to the micro-grids

Fig. 3: Real-time supply and demand to the micro-grids

supply, and vice versa. We also show the changes in *roaming-price* for each micro-grid in Figure 4(b), as calculated from Equation (16). The values of the pre-determined constants are as follows:  $a = 0.001$ ,  $b = 0$ , and  $c = 0$ . Additionally, We also show the price difference between *home* and *roaming* price in different time instant throughout the simulation in Figure 4(c). We see that for all micro-grids, *roaming-price* is greater than the *home-price*. Therefore, each micro-grid maintains the pricing policy discussed in Section IV-B.

Figure 5 shows the comparison of *home* and *roaming* prices for PHEVs charging using D2P (proposed), D2R, UDP, QCF, and static pricing policies. We see that with the implementation of D2P, the cost to consume energy with *home-price* and *roaming-price* is less than that with the other pricing policies, as discussed in Section VI-B. So, the proposed approach for distributed pricing policy gives better result than the centralized one. In the proposed scenario, we see that a fair pricing policy is also maintained, i.e., the PHEVs charge their batteries from their home micro-grid by paying less price than that of the foreign PHEVs.

The total energy requests for each PHEV is shown in Figure 6(a) during the simulation time. In Figure 6(c), we evaluate the delay of charging the batteries for each PHEV as depicted in Section VI-C. We also calculate the total energy discharged by each PHEVs, as discussed in Section III-B. Figure 6(b) shows the total discharged energy for each PHEVs. Therefore, in the smart grid architecture, PHEVs play an important role to balance the supply-demand curve while micro-grids do not have sufficient energy to provide services to all customers. Intuitively, we can say that with more numbers of PHEVs, micro-grids can provide reliable services to the customers.

The total charging cost for PHEVs is shown in Figure 7(a)

<sup>2</sup><http://www.eia.gov/electricity/monthly/index.cfm>

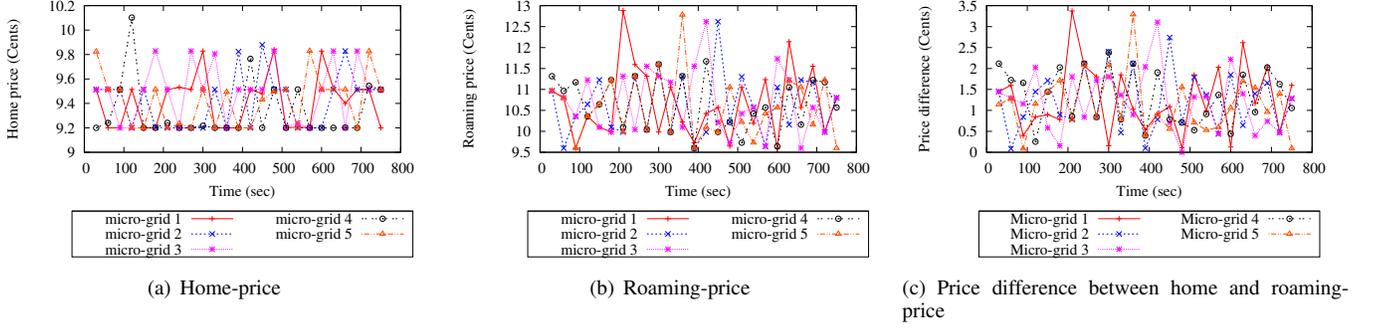


Fig. 4: Real-time price and difference between home and roaming-price for each micro-grid

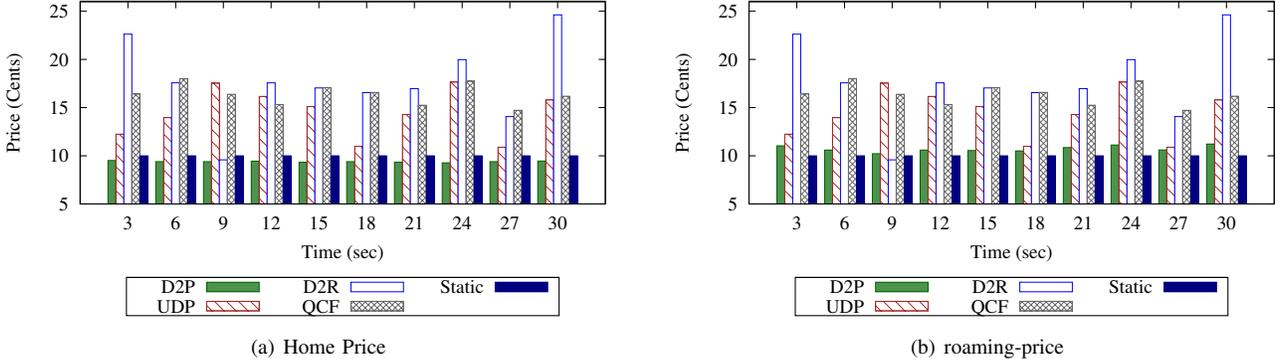


Fig. 5: Comparison of home and roaming-price

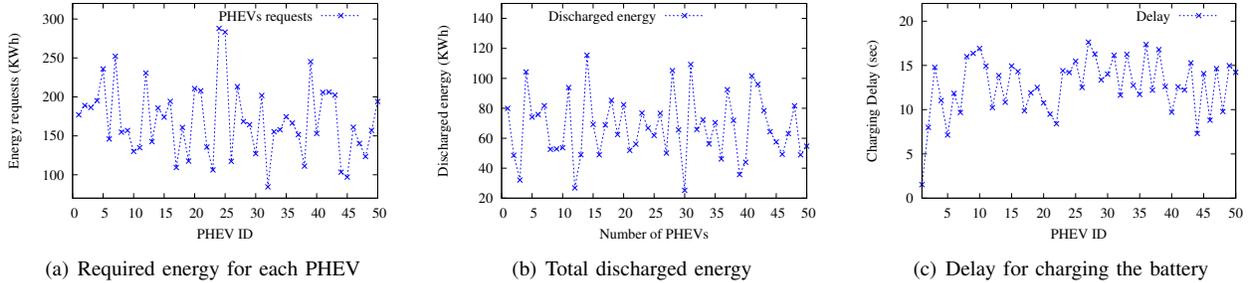


Fig. 6: Charged and discharged energy of PHEVs, and charging delay

using D2P (proposed), D2R, UDP, QCF, and static pricing policies corresponding to the energy requests from PHEVs, as shown in Figure 6(a). We see that with an increase in the number of PHEVs, D2P outperforms over the implementation of the D2R, UDP, QCF, and static scenarios. PHEVs charge their batteries from the *home micro-grid* according to the *home-price*,  $p_h^t$ , and from the *foreign micro-grid*, according to the *roaming-price*,  $p_r^t$ .

In Figure 7(b), total utility for PHEVs is shown while charge. With the implementation of distributed pricing policy (D2P), the overall utility of the PHEVs increases 34.26%, 41.87%, 44.33%, and 13.51% than over using D2R, UDP, QCF, and static pricing policy, respectively. Additionally, we also calculate total utility for PHEVs while discharge their

energy with roaming-price and real-time price over the static price. The utility with roaming-price illustrates that the PHEVs discharge their energy while they are paid according to the roaming-price. On the other hand, utility with real-time price illustrates that the PHEVs discharge their batteries with real-time price. We see that total utility of the PHEVs increases with an increase in the number of PHEVs. Therefore, using D2P scheme, utility for PHEVs increases both in charging and discharging processes.

## VII. CONCLUSION

In this paper, we proposed a distributed dynamic pricing policy for PHEVs management in smart grid which considers real-time supply and demand to the micro-grids. We used linear optimization approach to decide the real-time price,

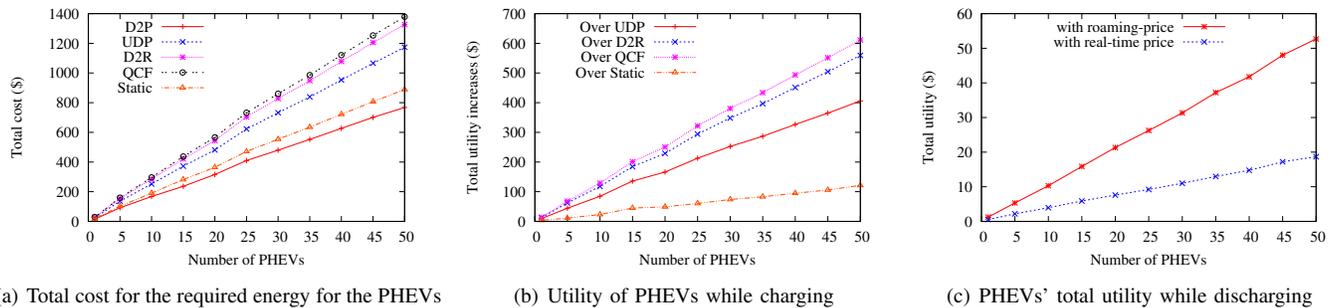


Fig. 7: PHEVs' charging cost and utility

and a multi-attribute decision process to maximize the utility for PHEVs. The real-time price is presented as — *home* and *roaming*. A fair pricing policy is also maintained with the implementation of the distributed one, i.e., the home PHEVs consume energy with *home-price*, and foreign PHEVs consume energy with *roaming-price*, which is higher than or equal to the *home-price*. Thus, home users do not have to pay higher cost due to the presence of more number of foreign PHEVs. Through simulations, we showed that the proposed approach performs better than the centralized one. The overall utility for PHEVs increases 34.26%, 41.87%, 44.33%, and 13.51% than that using D2R, UDP, QCF, and static pricing policy, respectively. We also showed the delay for charging the battery for each PHEV. Additionally, we showed that PHEVs' utility increases for discharging as well.

The future extension of this work will involve the improvement of the utility for both the grid and PHEVs in order to have more reliable, and cost-efficient energy management in smart grid, while incorporating cooperation among PHEVs to reduce the communication cost, and evaluating the performance of the proposed pricing scheme, D2P, with real traces for PHEVs. We also plan to incorporate the remote (wireless) charging and discharging [25] mechanism, so that cost to travel the charging/discharging station can be minimized.

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